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Reversion of turbulent to laminar flow

By V. C. PATEL AND M. R. HEAD
Cambridge University Engineering Department

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It has been shown experimentally that quite large departures occur from the universal inner-law velocity distribution in the presence of severe favourable pressure gradients in turbulent boundary layers and that these departures are associated with the tendency for the turbulent boundary layer to revert to a laminar state. From the measurements a criterion for the onset of reverse transition has been deduced in terms of the mean shear-stress gradient in the wall region of the flow. Experiments in fully developed pipe and channel flows suggest that the proposed criterion may be quite generally applicable to all fully turbulent shear flows.

1. Introduction

In experiments described by the first author (Patel 1965*a*) turbulent boundary-layer velocity profiles were measured in favourable and adverse pressure gradients of varying severity. Typical results are shown in figure 1, from which it will be seen that, in favourable pressure gradients, quite major departures occurred from the so-called universal inner-law velocity distribution. It appeared that, as a result of the favourable pressure gradient, the viscous sublayer had increased substantially in thickness whilst the velocity defect in the outer part of the layer had decreased. From the general appearance of these profiles and comparison with the measurements of Schlinger & Sage (1953) shown in figure 2 it seemed likely that the layer was reverting to the laminar state.

It is now well known that reverse transition may occur in wind-tunnel contractions, turbine nozzle cascades (Senoo 1957) and supersonic nozzles (Sergienko & Gretsov 1959), but at the time of beginning the present work it appeared that no systematic investigation of the phenomenon had been carried out. Since then papers by Launder (1964), Launder & Stinchcombe (1967), Badrinarayanan (1966) and Badrinarayanan & Ramjee (1968) have appeared which describe investigations broadly similar to that reported here but devoted to rather different aspects of the problem. The various investigations are in fact largely complementary; Launder's and Badrinarayanan's observations, which include turbulence measurements as well as measurements of mean velocity profiles, cover the complete reverse transition process while the present investigation is substantially confined to the initial stages, the object being to define as closely as possible the conditions leading to the breakdown of the fully developed turbulent flow.

There is of course no reason to expect that any quantitative relationship should exist between conditions defining the onset of instability in a laminar flow and conditions defining the lowest level at which the corresponding fully developed turbulent flow can be maintained. In other words, we should expect quite different considerations to govern the onset of normal and reverse transition; the former will depend upon the characteristics of the laminar flow (and

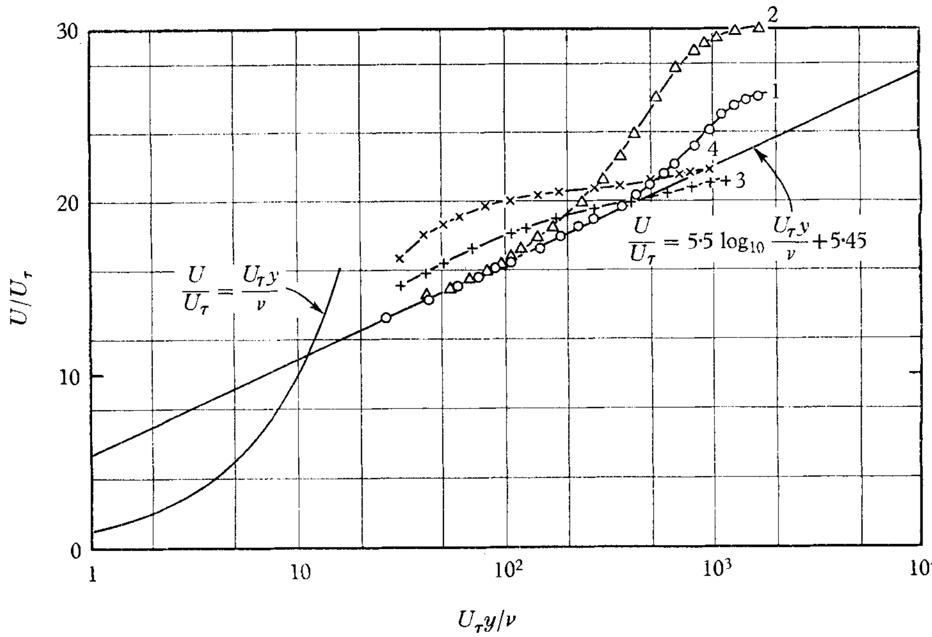


FIGURE 1. Blower tunnel results. ○, Zero pressure gradient; △, mild adverse gradient; ×, severe favourable gradient; +, moderate favourable gradient (round pitot diameter 0·054 in., Macmillan's (1957) corrections).

	R_θ	H	Δ_p
○	4500	1·382	0
△	4125	1·540	0·0121
×	785	1·359	−0·0236
+	1140	1·313	−0·0117

of any disturbances that may be present), whilst the latter will depend upon the characteristics (normally quite well defined) of the fully developed turbulent flow.

In the past it appears to have been generally accepted that it is the Reynolds number of a boundary layer or channel flow which determines whether or not the flow can exist in a self-maintaining turbulent condition. The evidence presented here, however, does not support this supposition, and it is in fact established by experiment that the initiation of reverse transition in a boundary layer by a sufficiently strong favourable pressure gradient is substantially independent of the boundary-layer Reynolds number. This result suggests further that even in the case of pipe or channel flows, where the importance of Reynolds number has been clearly established, reversion to laminar flow occurs not as a direct result of the reduction of Reynolds number but as a consequence of the increasingly favourable pressure gradient (expressed in suitable non-dimensional terms) that this reduction of Reynolds number represents. This suggestion is supported by existing measurements in pipe flow, but further experiments were performed to establish that the proposed criterion for reverse transition is universally valid for boundary-layer and channel flows.

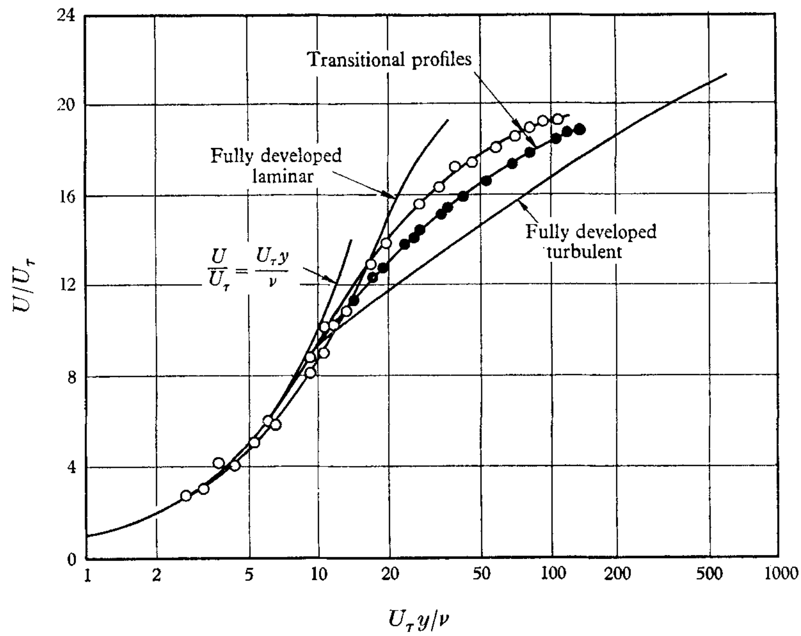


FIGURE 2. Velocity profiles measured by Schlinger & Sage (1953) in flow between parallel plates.

2. A re-examination of previous measurements

As reported earlier (Patel 1965*a*) the profiles shown in figure 1 were measured in a wind tunnel with a flexible wall and skin-friction values were obtained by the use of a sublayer fence approximately 0·003 in. high. This was calibrated *in situ* by means of a Preston tube in substantially zero pressure gradient. As already mentioned it seemed likely that the major departures from the inner-law observed in favourable pressure gradients were associated with reversion to laminar flow, and it seemed likely at this stage that the relatively low boundary-layer Reynolds numbers attainable in the tunnel with strong favourable gradients were responsible for this behaviour.

Further experiments were therefore carried out in the entry length of an 8 in. diameter pipe fitted with a centre-body. Measured velocity profiles obtained at two different positions relative to the centre-body and for two different entry lengths are shown in figure 3. In this figure, as in figure 1, the severity of the pressure gradient as it affects the flow in the wall region is indicated by the parameter $\Delta_p(= [\nu/\rho U_\tau^3]\, dp/dx)$ which is formed from the pressure gradient and the natural scales of velocity (U_τ) and length (ν/U_τ) in this region.

The coincidence of the profiles for $\Delta_p = -0{\cdot}0203$ and $\Delta_p = -0{\cdot}0235$ at the downstream station suggests strongly that it is the favourable pressure gradient and its effect on the wall region of the flow which is primarily responsible for departures from the inner-law (and, by inference, for the initiation of reverse transition), and that the boundary-layer Reynolds number, which differs by a factor of almost 3 in the two cases, is of minor importance. It will be noted also that the values of the form parameter H and the pressure gradient parameter $(\theta/U_1)\,dU_1/dx$ differ markedly in the two cases.

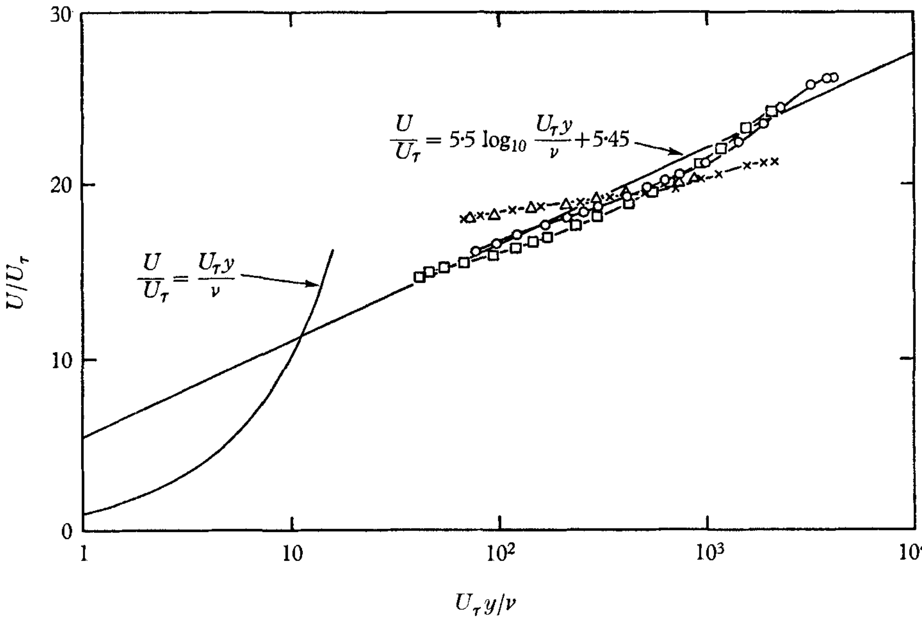


FIGURE 3. Favourable-pressure-gradient profiles.

Δ_p			R_θ	H
$-0{\cdot}01975$	□	Entry 124 in.	5480	1·323
$-0{\cdot}01239$	△	Entry 124 in.	9640	1·312
$-0{\cdot}0203$	○	Entry 22 in.	750	1·545
$-0{\cdot}0235$	×	Entry 124 in.	2220	1·264

This result was unexpected, but on reflexion it is not unreasonable, since most of the production and dissipation of turbulence energy takes place in the wall region, where conditions are largely independent of the properties of the outer turbulent flow. This region of the flow is normally characterized by near equilibrium of the production and dissipation (Townsend 1961) of turbulent energy and it is reasonable to infer that the initiation of reverse transition represents the breakdown of this equilibrium.

We may digress here to note that for the zero-pressure-gradient boundary layer the flow within the equilibrium region close to the wall is characterized by a nearly constant shear stress and a semi-logarithmic velocity distribution given by the well-known inner-law

$$U^* = \frac{1}{K}\ln y^* + B, \tag{1}$$

where $U^* = U/U_\tau$, $y^* = U_\tau y/\nu$, U is the mean velocity, y the distance from the surface, $U_\tau = (\tau_w/\rho)^{\frac{1}{2}}$, τ_w the wall shear stress, ρ and ν are fluid density and kinematic viscosity respectively, and K and B are universal constants which take the numerical values 0·418 and 5·45 respectively, as suggested previously by Patel (1965*a*). For non-zero pressure gradients, the stress distribution in the wall region can no longer be treated as constant and Townsend (1962) suggests a linear approximation of the form

$$\tau = \tau_w + \alpha y, \tag{2}$$

where the shear-stress gradient α is a function of the pressure gradient and local flow accelerations. From considerations of the turbulent energy equation, Rotta (1951) and Townsend (1961) have suggested that substantial equilibrium exists between the production and dissipation of turbulent energy in the fully turbulent part of the wall region, and have shown that the usual mixing-length relation may be used to predict the mean-velocity distribution. Patel (1965*b*) has shown that the use of the slightly modified mixing-length relation

$$\frac{\partial U}{\partial y} = \frac{1}{Ky}\left(\frac{\tau}{\rho}\right)^{\frac{1}{2}}\left\{1 - \beta\frac{y}{\tau}\left|\frac{\partial \tau}{\partial y}\right|\right\} \tag{3}$$

due to Townsend, the assumption of a linear shear stress distribution, and Mellor's (1966) continuous eddy viscosity model, to provide a smooth transition from the fully turbulent region to the more general sublayer velocity distribution

$$U^* = y^* + \tfrac{1}{2}\Delta_p y^{*2}, \tag{4}$$

leads to the velocity distribution of the form

$$U^* = \frac{1}{K}\left\{\ln \pm \left[\frac{4}{|\Delta_\tau|}\frac{(1+\Delta_\tau y^*)^{\frac{1}{2}} - 1}{(1+\Delta_\tau y^*)^{\frac{1}{2}} + 1}\right] + K(B + 3{\cdot}7\Delta_p) + 2(1-\beta)\left[(1+\Delta_\tau y^*)^{\frac{1}{2}} - 1\right]\right\}, \tag{5}$$

where $\Delta_\tau = \nu\alpha/\rho U_\tau^3$ is the non-dimensional shear stress gradient analogous to the non-dimensional pressure gradient

$$\Delta_p = \frac{\nu}{\rho U_\tau^3}\frac{dp}{dx}.$$

The term involving the constant β in (3) was introduced by Townsend to take into account the diffusion of turbulent energy resulting from slight departures from the assumed equilibrium between production and dissipation. Townsend estimated the value of β to be in the region of 0·18 though, more recently, Bradshaw (1966) has put forward evidence which suggests a much smaller value. The positive sign in the logarithmic term refers to $\Delta_\tau > 0$, i.e. adverse pressure gradients, and the negative sign refers to $\Delta_\tau < 0$, i.e. favourable pressure gradients. Equation (5) is compared with the measurements of Newman (1951) in figure 4, the values of Δ_τ being obtained from best linear fits to his measured shear-stress distributions in the wall region. It will be seen that the agreement is excellent.

From the above comparisons in an adverse-pressure-gradient boundary layer we can conclude that: quite large departures from the usual semi-logarithmic law can occur in the presence of large adverse pressure gradients; we may expect similar departures in the opposite sense for large favourable pressure gradients; equation (5) can be used to describe quite accurately the velocity distribution in the fully turbulent part of the wall region. We shall make use of (5) in the subsequent work to test for the existence of fully turbulent flow and its breakdown during the process of reverse transition. In fact, (5) suggests that the criterion

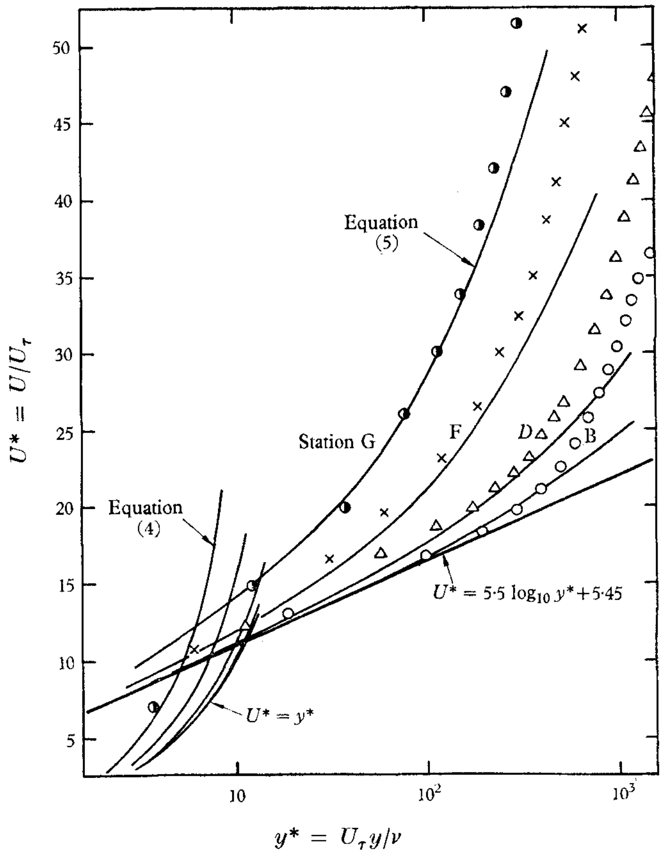


FIGURE 4. Wall region velocity profiles in Newman's (1951) experiments compared with equation (5).

	Δ_p	Δ_τ	Δ_τ/Δ_p
B	0·0094	0·0045	0·48
D	0·0279	0·0141	$0{\cdot}50_5$
F	0·1127	0·075	$0{\cdot}66_5$
G	0·3425	0·250	0·73

for the breakdown of fully developed turbulent flow must be sought in terms of the shear-stress gradient parameter Δ_τ. It is shown later that, for values of this parameter beyond a certain critical value, fully developed turbulent flow cannot be maintained and the flow begins to revert to the laminar state.

From previous observations in a favourable-pressure-gradient boundary layer and the considerations just outlined it may tentatively be concluded that major departures from the inner-law velocity distribution and the initiation of reverse transition occurred as a result of the large shear-stress gradients accompanying severe favourable pressure gradients. Since the early measurements were made at isolated points it was not possible to determine the exact location at which reverse transition was initiated. Further experiments were therefore carried out to establish more accurately the criterion for the breakdown of fully turbulent flow and to confirm that the onset of laminar reversion was independent of the overall boundary-layer characteristics such as R_θ and H.

3. Boundary-layer development in a strong favourable pressure gradient

3.1. *Apparatus and measurements*

The centre-body described in detail by Patel (1965*a*) was again used to produce favourable pressure gradients in the entry length of the 8 in. diameter pipe. This arrangement was adopted since the boundary-layer Reynolds number could be

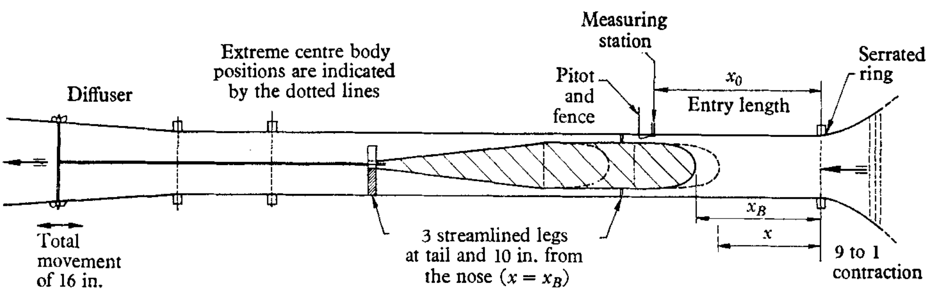


FIGURE 5. Experimental arrangement for boundary-layer development in a favourable pressure gradient.

readily varied by changing the length of the pipe upstream of the measuring station. The centre-body was mounted symmetrically in the pipe by means of six streamlined legs instead of the six piano wires used previously. The apparatus is shown schematically in figure 5. It will be seen that the position of the centre-body relative to the fixed measuring station could be varied without disturbing the working section of the pipe. The total travel of the body was such that its nose remained within 8·5 in. of the measuring station.

With an entry length of $x_0 = 124$ in. measurements of static pressure distributions along the pipe wall, and velocity profiles and skin friction at the measuring station ($x = x_0$) were made for various positions of the centre-body in the range $-7 < (x_0 - x_B) < 8{\cdot}45$ in.; the notation is illustrated in figure 5. The mass flow rate through the pipe was kept constant throughout. The velocity profiles were obtained by means of a flattened Pitot. Some of the measured pressure distributions are shown in figure 6. As expected, these remain unchanged in shape but are displaced in the x-direction by distances equal to the displacement of the centre-body. Thus, the only effect of the centre-body movement is to change the length over which the boundary layer develops in substantially zero pressure gradient. Effectively, therefore, the centre-body may be regarded as fixed and the position of the measuring station as varying. The variable $(x_0 - x_B)$ indicating the displacement of the centre-body can then be replaced by the streamwise distance $(x - x_0)$.

The pressure gradient at the measuring station was determined for each centre-body position by graphical differentiation of the corresponding pressure distribution. The readings of the skin-friction fence were corrected for pressure

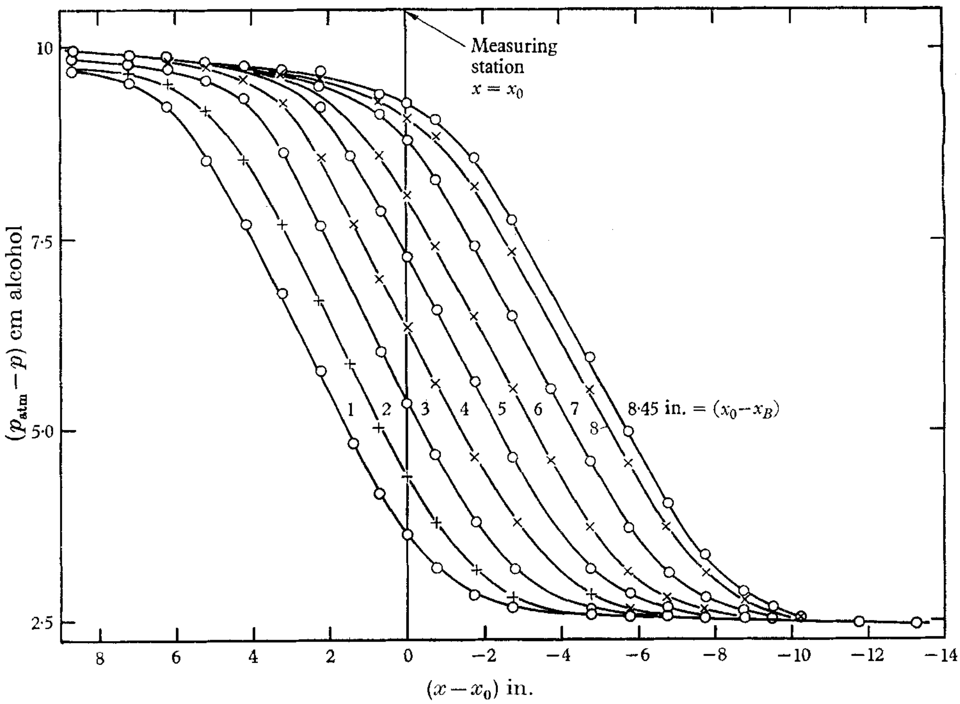


FIGURE 6. Typical pressure distributions on pipe wall. Entry length, $x_0 = 124$ in.

gradient effects using the procedure described by Patel (1965*a*). Some of the measured velocity profiles have been plotted on natural scales in figure 7. Figure 8 shows the profiles plotted in the usual inner-law co-ordinates. The overall boundary-layer characteristics H and R_θ are shown in figure 9 and the variation of the pressure gradient parameter Δ_p is shown in figure 10.

A similar set of measurements was made with the entry length reduced to $x_0 = 22$ in. The pressure distribution showed exactly the same trends as before. The mass flow rate was kept constant at the value which gave the same value of the parameter Δ_p at $(x_0 - x_B) = 2$ in. as with the 124 in. entry. This was done to ensure that the values of the pressure gradient dp/dx and the wall shear stress τ_w were not widely different in the two cases, so that the fence calibration curve obtained in fully developed pipe flow did not require any extrapolation. The measured velocity profiles have been plotted in figure 8. The corresponding values of H and R_θ are shown in figure 9 and the variation of Δ_p is shown in figure 10.

3.2. *Results and discussion*

The velocity profiles shown in figures 7 and 8 confirm the earlier observation that in favourable pressure gradients the viscous sublayer increases in thickness and the velocity defect in the outer region decreases. The present measurements of

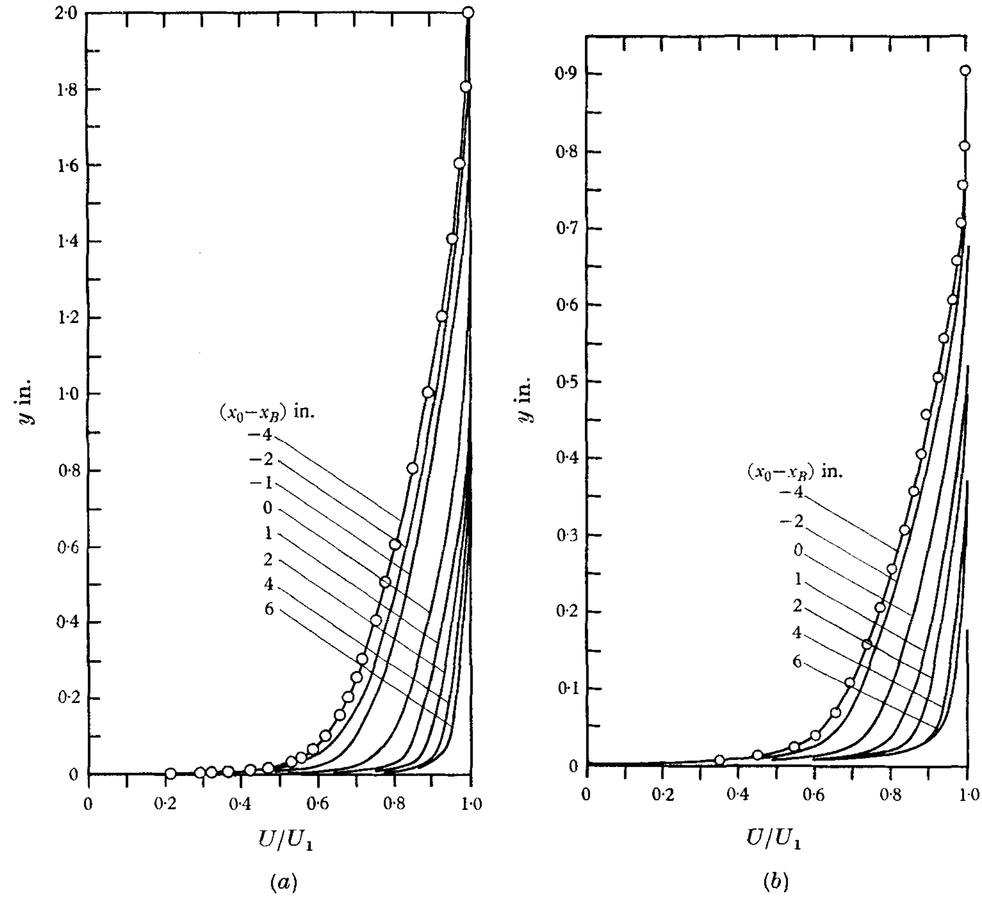
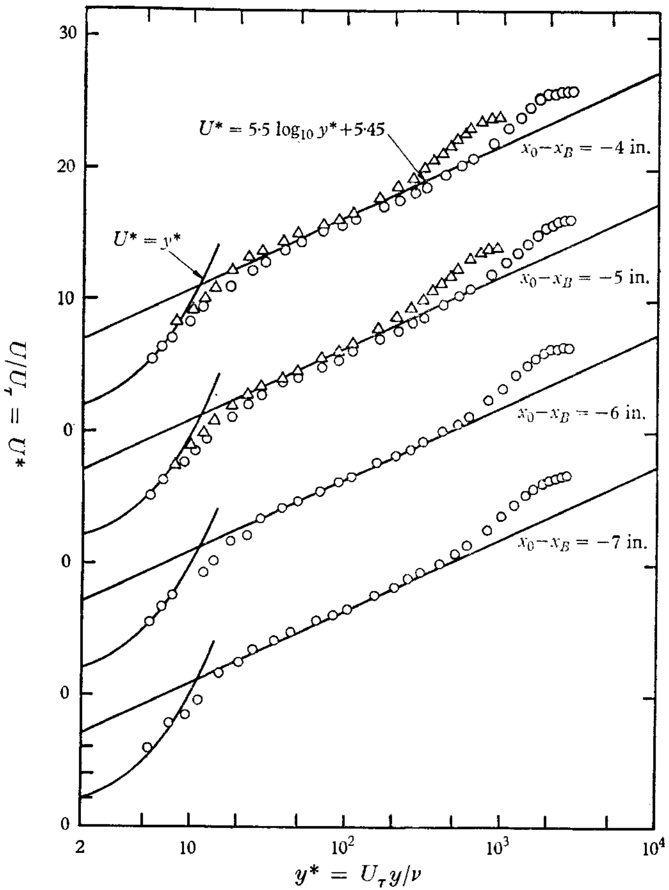


FIGURE 7. (*a*) Velocity profiles, $x_0 = 124$ in. entry. (*b*) Velocity profiles, $x_0 = 22$ in. entry.

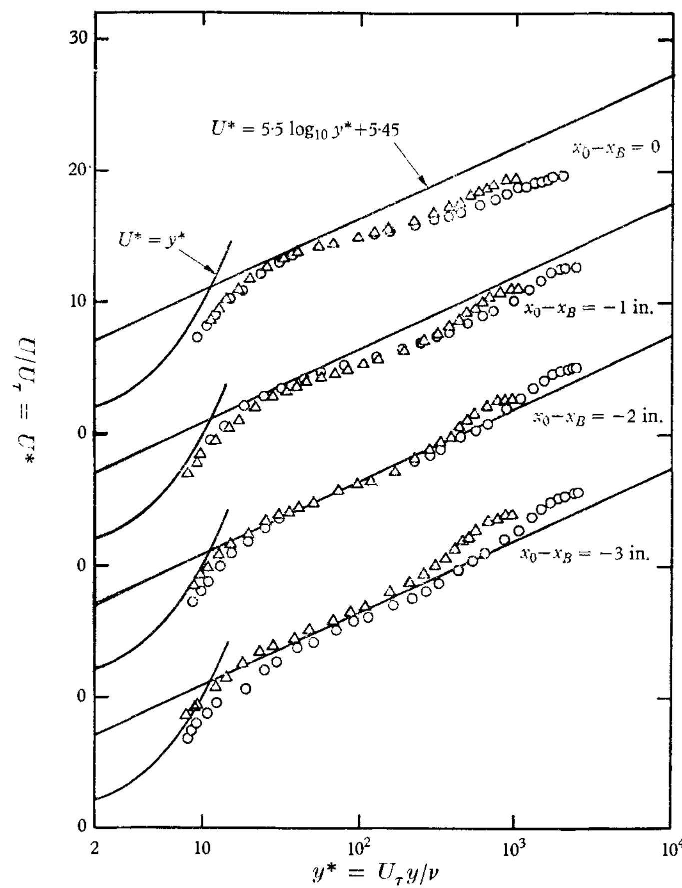
the wall-region velocity distributions are particularly useful since the wall shear stress has been measured directly by a method which is both reliable and accurate. The inner-law plots of figure 8 suggest the existence of three distinct régimes of flow. These may be interpreted as follows.

(i) In the region $-7 < (x_0 - x_B) < -3$ in., where the pressure gradient parameter is $0 > \Delta_p > -0{\cdot}0034$, no significant departures from the usual semi-logarithmic law of the wall are observed. The pressure gradient, as it affects the wall region, may therefore be regarded as small. It will be recalled that a similar range of Δ_p was suggested for the use of Preston tubes in favourable-pressure-gradient boundary layers (Patel 1965*a*). For such pressure gradients there is a



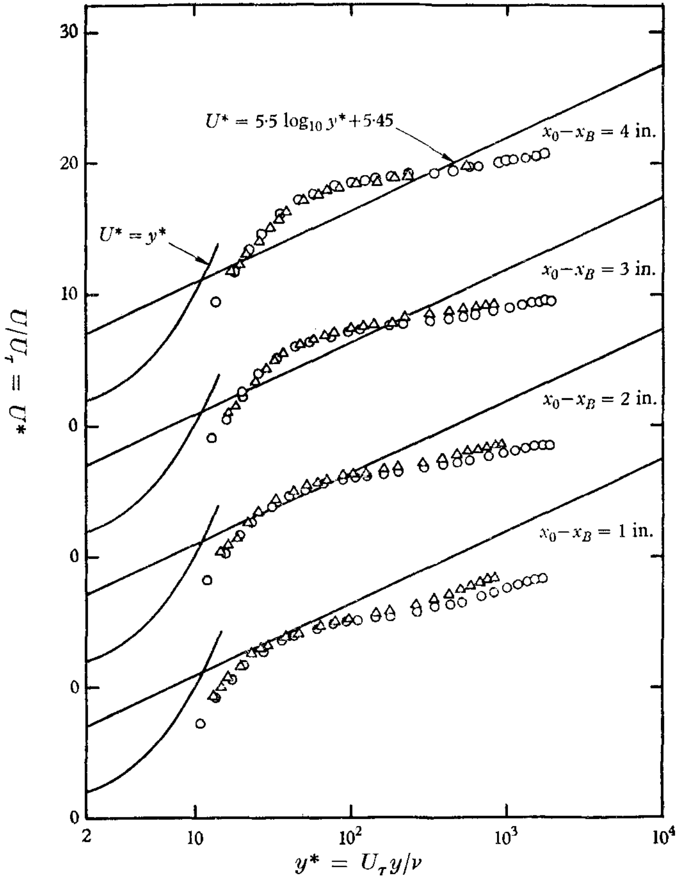
(*a*) Velocity profiles in wall co-ordinates. △, 22 in. entry; ○, 124 in. entry.

$x_0 - x_B$ (in.)	△ U_1 (ft./s)	△ c_f	○ U_1 (ft./s)	○ c_f
−7	—	—	51·71	0·00279
−6	—	—	51·58	0·00284
−5	47·35	0·00344	51·36	$0{\cdot}00290_5$
−4	47·63	0·00345	51·60	$0{\cdot}00294_5$



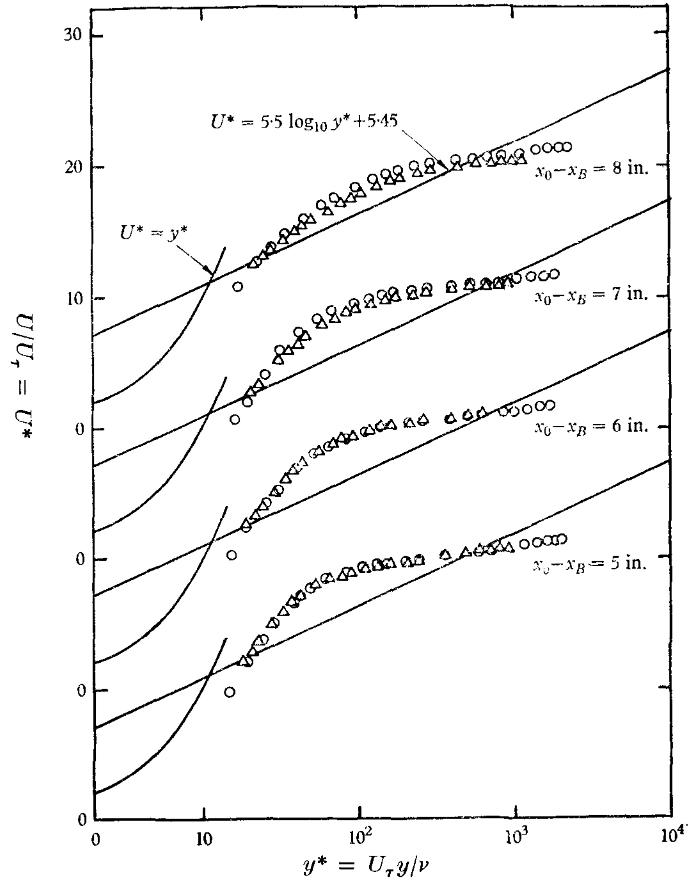
(*b*) Velocity profiles in wall co-ordinates. △, 22 in. entry; ○, 124 in. entry.

$x_0 - x_B$ (in.)	△ U_1 (ft./s)	△ c_f	○ U_1 (ft./s)	○ c_f
−3	47·74	0·00351	51·87	$0{\cdot}00305_5$
−2	48·65	0·00386	52·41	0·00318
−1	50·02	0·00449	52·97	0·00387
0	53·98	0·00527	54·24	0·00516



(c) Velocity profiles in wall co-ordinates. △, 22 in. entry; ○, 124 in. entry.

$x_0 - x_B$ (in.)	△ U_1 (ft./s)	△ c_f	○ U_1 (ft./s)	○ c_f
1	59·65	0·00590	59·72	0·00597
2	68·91	0·00578	68·32	0·00578
3	77·93	0·00537	78·65	0·00521
4	86·18	0·00492	87·32	0·00467



(d) Velocity profiles in wall co-ordinates. △, 22 in. entry; ○, 124 in. entry.

$x_0 - x_B$ (in.)	△ U_1 (ft./s)	△ c_f	○ U_1 (ft./s)	○ c_f
5	93·05	0·00461	$94{\cdot}96_5$	0·00435
6	101·00	0·00440	101·0	$0{\cdot}00421_5$
7	104·7	0·00444	106·3	0·00416
8	109·1	0·00475	110·0	0·00432

FIGURE 8(c) AND (d)

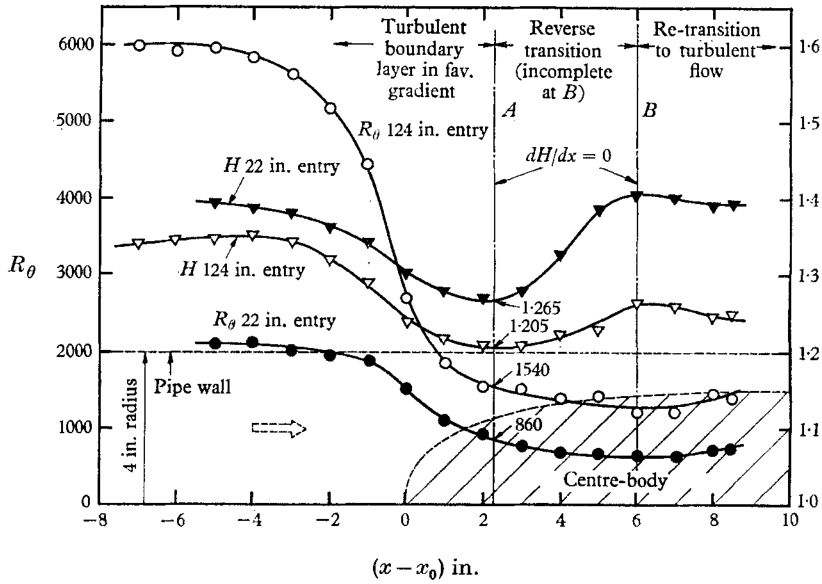


FIGURE 9. Boundary-layer development in 8 in. diameter pipe.

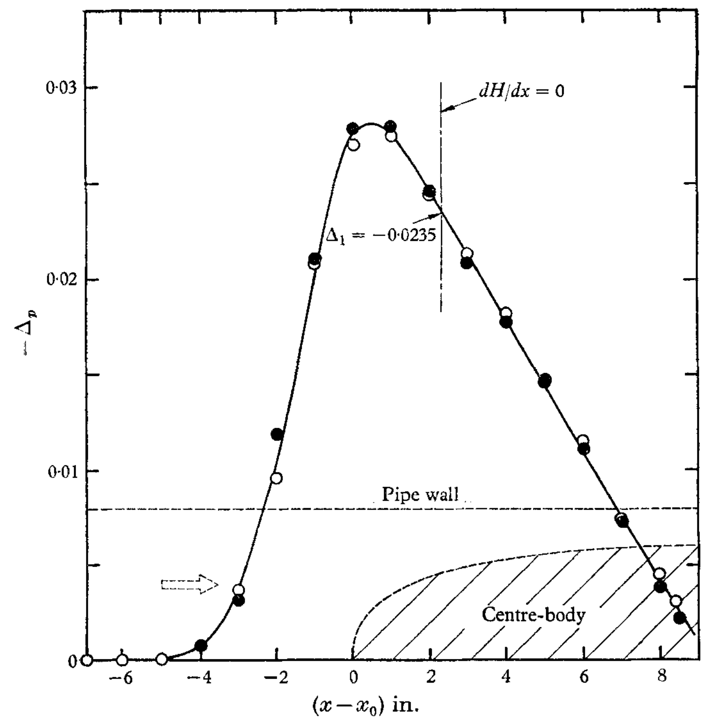


FIGURE 10. Variation of the pressure gradient parameter Δ_p. ○, x_0 = 124 in. entry; ●, x_0 = 22 in. entry.

fully turbulent region near the wall in which there is energy equilibrium and the mean-velocity distribution is given by mixing-length theory with constant shear stress.

(ii) In the region $-3 < (x_0 - x_B) < 2$ in., where $-0{\cdot}0034 > \Delta_p > -0{\cdot}0245$, the velocity measurements fall below the semi-logarithmic straight line and show the trends predicted by (5). The boundary layer is still fully turbulent and the wall region is still characterized by energy equilibrium.

(iii) Finally, in the region $2 < (x_0 - x_B) < 8$ in., the velocity profiles lie above the straight line and indicate trends similar to those observed by Patel & Head (1968) and Schlinger & Sage (1953) (figure 2) in transitional flows in pipes and channels. These profiles characterize a boundary layer which is reverting from the turbulent to the laminar state. Equation (5) can no longer be applied in the wall region and equilibrium between production and dissipation of turbulence energy is no longer maintained. Major departures from (5) are first observed at $(x_0 - x_B) = (x - x_0) = 2$ in., where $\Delta_p = -0{\cdot}0245$ and $\Delta_\tau = -0{\cdot}009$, this latter being evaluated by matching the measured mean velocity profiles with (5). The reverse transition process is therefore initiated somewhere in this region for both entry lengths.

Figure 9 shows the reduction in the boundary-layer Reynolds number, R_θ, brought about by the favourable pressure gradient. With the 124 in. entry length the initial R_θ of about 6000 in zero pressure gradient is reduced to about 1210 at $(x - x_0) = 6$ in. The corresponding reduction for the 22 in. entry is from 2100 to 615. However, the most important observation to be made from figure 9 is the behaviour of the shape factor H, which starts from values corresponding to the flat plate boundary layer, reaches a minimum and then rises again. For both entry lengths the minimum value of H occurs at $(x - x_0) = 2{\cdot}3$ in., where Δ_p, from figure 10, is $-0{\cdot}0235$ and Δ_τ, as estimated from figure 8, is in the region of $-0{\cdot}009$. This position corresponds almost exactly with that where departures of the velocity profiles from (5) were first observed. This was to be expected since, for a turbulent boundary layer in continuously favourable pressure gradient (as is obtained in the present experiments) H is expected to decrease, but, if at some stage the boundary layer goes through a reverse transition, H must rise asymptotically to its laminar value, which is in the region of 2 to 3. The point where a minimum in H occurs must therefore correspond closely to the position where laminar reversion is initiated. This observation has also been made by Launder (1964).

It is seen from figure 10 that the variation of Δ_p is almost identical for the two entry lengths even though the mass flow was adjusted to obtain the same value of Δ_p only at $(x - x_0) = 2$ in. Since Δ_p is essentially a wall-region parameter, this suggests that the flow near the wall is not affected by the state of the boundary layer upstream of the region of influence of the centre-body or by the local values of the overall boundary-layer characteristics such as H and R_θ. This is further confirmed by the coincidence of the velocity distributions (shown in figure 8) in the neighbourhood of the wall. The point at which the breakdown of turbulent flow occurs, i.e. where H reaches a minimum and velocity profiles depart from the inner-law in the form of (5), is also independent of R_θ. It will be noted that for the two entry lengths the Reynolds numbers where laminar reversion begins ($R_\theta = 1540$ and 860) differ by a factor of almost 2. Velocity profiles shown in figure 8 suggest that the breakdown of turbulent flow is initiated in the wall region of the boundary layer, and the parameter governing this breakdown must therefore be that characterizing this region. Since the shear-stress gradient parameter Δ_τ is found to describe the flow in the fully turbulent part of the wall region quite adequately, we conclude that there is a critical value of Δ_τ in the region of $-0{\cdot}009$ beyond which fully turbulent flow cannot be maintained; flows in which $\Delta_\tau < -0{\cdot}009$ cannot be fully turbulent and must be transitional. This is the proposed criterion for the initiation of reverse transition.

So far, the discussion has been limited to the behaviour of a turbulent boundary layer in a favourable pressure gradient, and the conditions under which reverse transition is initiated. It is clear that the transitional flow beyond this point cannot be analyzed by the existing theories for either turbulent or laminar boundary layers. In the present experiments the length over which a strong favourable pressure gradient could be maintained was limited, and consequently, as far as can be judged from measurements of mean quantities, a fully laminar state was not achieved. It will be seen from figure 9 that, as a result of a considerable reduction in the favourable gradient beyond $(x-x_0) = 5$ in., H begins to fall and R_θ begins to rise, indicating a re-transition of the partly laminar boundary layer towards the fully turbulent state. Velocity profiles at $(x-x_0) = 8$ in., as shown in figure 8, suggest that the boundary layer has not completely reverted back to turbulent. Although no experimental evidence is available on this point, it seems likely that, while the initiation of reverse transition is independent of the boundary-layer Reynolds number, the extent of the favourable pressure gradient required to complete the reversion process and achieve fully laminar flow must depend very much on this parameter.

Launder (1964) and Launder & Stinchcombe (1967) have measured the developments of boundary layers in strong favourable pressure gradients. These results do not in any way contradict the conclusions drawn from the present experiments, but their usefulness, in the present context of establishing a criterion for the onset of reverse transition, is rather limited since they suffer from two major drawbacks. First, the wall shear stress was not measured directly, and it is therefore impossible to plot the velocity profiles in the inner-law form and determine the position where departures from the velocity law of equation (5) first occurred. The values of the pressure gradient parameter Δ_p and the shear-stress gradient parameter Δ_τ also remain unknown. Secondly, in most of Launder's measured boundary-layer developments, the Reynolds numbers upstream of the pressure gradients were so low ($200 \leqslant R_\theta \leqslant 1500$) that under the pressure gradients R_θ dropped to values well below the 320 given by Preston (1958) as the minimum for a turbulent boundary layer in zero pressure gradient. These results cannot therefore be used to support the observation that the onset of reverse transition is independent of R_θ.

3.3. *Further remarks concerning boundary-layer measurements*

The present experiments have demonstrated quite conclusively that the onset of laminar reversion in a fully developed turbulent boundary layer occurs as a direct result of a particular shear-stress distribution in the wall region (which, in the present case, has been imposed by a favourable pressure gradient) and not, as is commonly supposed, as a result of the reduction in the overall Reynolds number of the flow. Experimental results suggest that a critical value in the region of $-0{\cdot}009$ for the shear-stress gradient parameter

$$\Delta_\tau \equiv \frac{\nu}{\rho U_\tau^3}\alpha$$

(where $\tau = \tau_w + \alpha y$ in the wall region) marks the boundary between fully developed turbulent flow and transitional flow; a fully turbulent flow begins to revert to the laminar state when $\Delta_\tau < -0{\cdot}009$. It will be recalled that the values of Δ_τ quoted so far have been inferred from comparisons of measured mean velocity profiles with (5). It would seem highly desirable to confirm these values by direct measurements of shear stress distributions through the boundary layer. Such measurements are, however, extremely difficult to make owing to the fact that favourable-pressure-gradient boundary layers are usually very thin and the region close to the wall, which is of immediate interest here, is smaller still. Judging from the success of mixing-length analysis in adverse pressure gradients, however, we may expect the values of Δ_τ deduced above to be fairly reliable.

Attempts at defining the conditions leading to the onset of reverse transition in a turbulent boundary layer have also been made by Launder (1963, 1964), Launder & Stinchcombe (1967), Back, Massier & Gier (1964), Moretti & Kays (1965) and Schraub & Kline (1965). These authors have sought the criterion in terms of a critical value of the pressure gradient parameter

$$K \equiv \frac{\nu}{U_1^2}\frac{dU_1}{dx} \equiv -\frac{\nu}{\rho U_1^3}\frac{dp}{dx}$$

or various combinations of it with the skin-friction coefficient in the form K/c_f, $K/c_f^{\frac{1}{2}}$, and $K/c_f^{\frac{3}{2}}$. It will be noted that this last parameter is identical with the pressure gradient parameter Δ_p defined earlier as $(\nu/\rho U_\tau^3)\,dp/dx$. While such local parameters give some indication of the state of the flow in the wall region it will be clear from the earlier discussion that the parameter K is not capable of specifying completely the flow in the wall region and we must therefore use a parameter such as Δ_τ (even though it may be difficult to estimate its value in a given flow situation) as the criterion governing the onset of reverse transition. It is of interest to note, however, that the critical value of K suggested by Schraub & Kline and verified by Moretti & Kays, namely $3{\cdot}5 \times 10^{-6}$, is in close agreement with the value $3{\cdot}7 \times 10^{-6}$ occurring in the present pipe and centre-body experiments at the point where laminarization is initiated. The measurements of Launder and those of Schraub & Kline do not, however, lend themselves to further analysis since the skin-friction values are not known accurately. In fact, the values of c_f quoted by Schraub & Kline (obtained by evaluating the slope of the velocity profiles close to the wall) appear to be too high by almost 50 %. This is evidenced by the fact that their measured velocity profiles, when plotted in the universal U/U_τ *vs.* $U_\tau y/\nu$ representation, lie well below the usual semi-logarithmic law even for the cases where the boundary layer is known to be transitional and for which the profiles are expected to lie well above the logarithmic law.

It was thought at this stage that the criterion for the onset of reverse transition proposed here should apply equally well to other flow situations in which reverse transition is known to occur, as, for example, in axisymmetric pipe flow, parallel flow in channels, radial flow between parallel disks and in boundary layers with suction. The two relatively simple cases of pipe and channel flows are examined in the next section and it is shown that the proposed criterion may be quite generally applicable.

4. Reversion to laminar flow in pipes and channels

4.1. *Pipe flow*

For fully developed flow in a pipe it is easy to show that the non-dimensional pressure gradient and shear-stress gradient parameters are given by

$$\Delta_p = 2\Delta_\tau = -\frac{4}{Re}\left(\frac{2}{c_f}\right)^{\frac{1}{2}}. \tag{6}$$

Here, the skin-friction coefficient c_f and the Reynolds number Re are based on the mean velocity $\overline{U}$ and the pipe diameter D. If the power-law relation

$$c_f = 0{\cdot}079(Re)^{-\frac{1}{4}} \tag{7}$$

is taken as appropriate to the lower range of Reynolds numbers for fully developed turbulent flow then

$$\Delta_p = 2\Delta_\tau = -20{\cdot}1(Re)^{-\frac{7}{8}}. \tag{8}$$

Thus, as the Reynolds number is reduced, the favourable pressure gradient, as it affects the flow in the wall region, effectively increases.

In order to clarify some of the points raised by the present study of reverse transition the authors found it necessary to undertake a detailed re-examination of the classical results of fully developed flows in circular pipes and also in parallel channels. This investigation has been reported in a separate paper (Patel & Head 1968) but the results which have a direct bearing on the present problem are reproduced in figures 11 and 12. Figure 11 shows the variation of skin-friction coefficient with Reynolds number in fully developed pipe flow. As expected, the measurements made with Reynolds number decreasing did not differ from those normally obtained with Reynolds number increasing, and from figure 11 it appears that departure from the skin-friction law for fully developed turbulent pipe flow, (7), first occurs at a Reynolds number in the region of 3000. As mentioned in the earlier paper, this result is in agreement with the observations of Binnie & Fowler (1947) and Lindgren (1957).

Taking the value $Re = 3000$ as defining the lower limit for which fully developed turbulent pipe flow is observed we see that this corresponds to

$$\Delta_p = -0{\cdot}018 \quad \text{and} \quad \Delta_\tau = -0{\cdot}009,$$

values which are very similar to those for which significant departures from the inner-law occurred in turbulent boundary-layer flow. In fact, it will be seen that the value of Δ_τ obtained above for pipe flow is in excellent agreement with that observed at the point of initiation of reverse transition in the boundary-layer experiments.

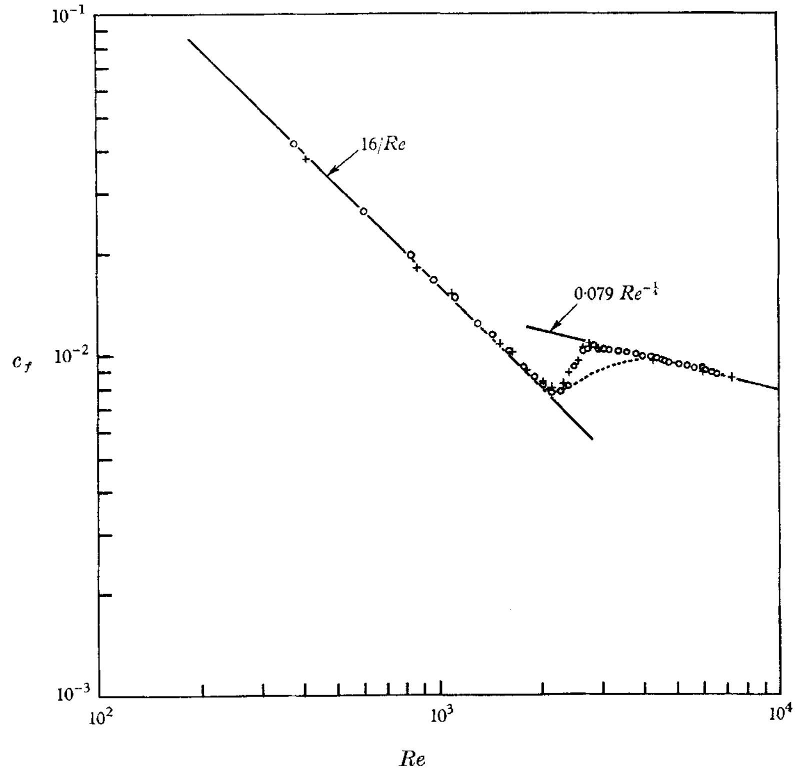


FIGURE 11. Skin friction in pipe flow. ○, $\frac{1}{4}$ in. diameter pipe; +, $\frac{1}{2}$ in. diameter pipe; – – –, Nikuradse

In the above comparison between the usual parallel pipe flow and a two-dimensional turbulent boundary layer it has been assumed that the flow at $Re = 3000$ in the pipe (obtained by decreasing the Reynolds number from some higher value) corresponds to the point where reverse transition is initiated in a boundary layer. This assumption cannot of course be substantiated with any great rigour owing to the fact that the two flows are basically different in many respects. For example, in pipe flow the reduction in Reynolds number is accomplished over a space of time and the fluid under examination at any fixed position along the pipe is changing constantly. Thus, the breakdown of the fully turbulent flow under the reduction of Reynolds number cannot strictly be considered as the decay of existing turbulence. (This is further supported by the apparent lack of hysteresis in pipe flows with specially designed smooth entries where laminar flow can be maintained for Reynolds numbers much greater than the lower critical value of about 1800.) In the case of the boundary layer, however, transition to laminar flow is accomplished over a physical distance by means of the favourable pressure gradient and there is a straightforward decay of turbulence in the same mass of fluid.

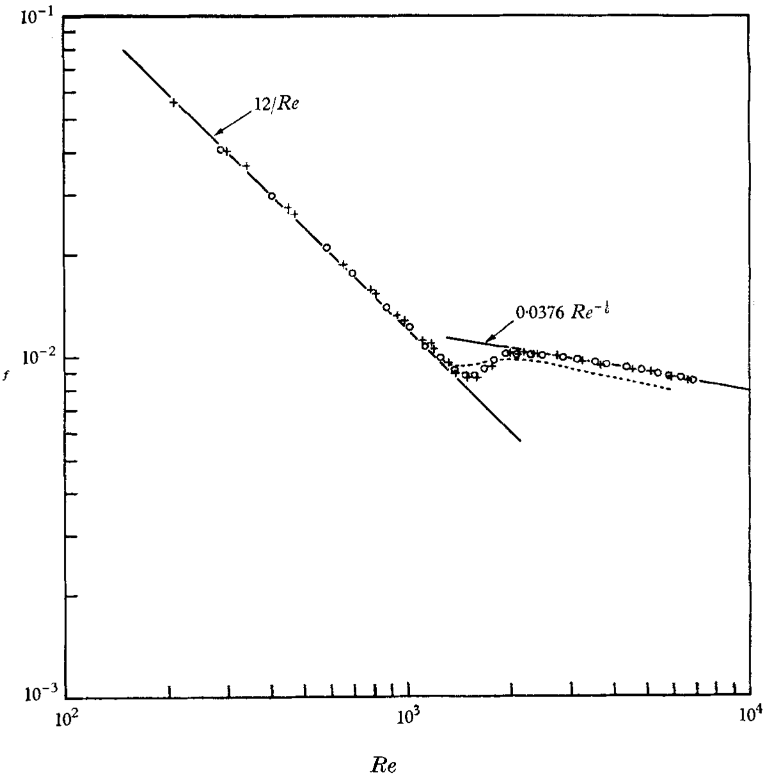


FIGURE 12. Skin friction in channel flow. O, Clear entry; +, $\frac{1}{8}$ in. diameter wire at entry; – – –, Davies & White (1928).

Conditions which are more obviously comparable to those occurring in a boundary layer undergoing reverse transition have been obtained in the experiments of Laufer (1962, p. 166) and Sibulkin (1962). These authors have studied the decay of fully developed pipe turbulence following gradual and abrupt transitions from one cylindrical pipe to another of larger diameter. In Laufer's experiment the Reynolds number decreased by a factor of 2·5 (from 3450 to 1380) while in Sibulkin's experiments the factor was 4·0 (*Re* decreasing from 2400 to 600, 4800 to 1200 and from 7200 to 1800). In all the cases studied the initially fully turbulent flow tended asymptotically to the laminar state, the distance within which the mean-velocity profile became parabolic depending upon the downstream Reynolds number. Since these investigations were concerned mainly with the decay of turbulence, these authors did not establish the flow of interest in the present study, namely the flow with minimum downstream Reynolds number which remained fully turbulent through the diffuser. There is, however, little doubt that this minimum downstream Reynolds number required to avoid reverse transition will be in the region of 3000, precisely the lowest value for which fully turbulent flow is observed in a straight pipe of constant diameter with disturbed entry conditions. Thus, the proposition made here that the values of Δ_τ in pipe flow and boundary-layer flow at the point of initiation of reverse transition are the same is confirmed.

4.2. *Channel flow*

For fully developed flow in a two-dimensional channel formed by two parallel flat plates it can readily be shown that the non-dimensional pressure gradient and shear gradient parameters are given by

$$\Delta_p = \Delta_\tau = -\frac{2}{Re}\left(\frac{2}{c_f}\right)^{\frac{1}{2}}. \tag{9}$$

Here, the skin-friction coefficient c_f and the Reynolds number Re are based on the mean velocity $\overline{U}$ and the distance between the parallel plates h. As pointed out recently by the present authors (Patel & Head 1968) this flow has not been studied as widely as the corresponding flow in circular pipes and consequently a well-established skin-friction law, such as (7) for pipe flow, cannot readily be found in the literature. From the recent results shown in figure 12 it becomes evident that in the lower range of Reynolds numbers for fully developed turbulent flow the skin-friction/Reynolds-number relation can be approximated by the power-law formula

$$c_f = 0{\cdot}0376(Re)^{-\frac{1}{6}}. \tag{10}$$

Combining equations (9) and (10) we have

$$\Delta_p = \Delta_\tau = -14{\cdot}6(Re)^{-\frac{11}{12}}. \tag{11}$$

Again, from an examination of figure 12 and some complementary evidence obtained by Patel & Head (1968), it is found that the lowest Reynolds number for which fully turbulent flow can be maintained in a channel is in the region of 3000. This corresponds to $\Delta_p = \Delta_\tau = -0{\cdot}0094$. This value of Δ_τ is in excellent agreement with those obtained from experiments in favourable-pressure-gradient boundary layers and in pipe flow.

The close correspondence in the value of Δ_τ for the different flows considered so far is quite remarkable in view of the fact that the shear stress gradient is related to local quantities such as the pressure gradient and flow accelerations in very different ways. This in itself suggests the importance of the parameter Δ_τ and tests the validity of the proposed criterion for the onset of reverse transition.

Reverse transition in channel flow has been studied recently by Badrinarayanan (1966). His experiments are very similar to those of Laufer and Sibulkin mentioned earlier; the width of a $\frac{1}{2}$ in. high channel was increased from 3 to 9 in. by means of a 48 in. long diffuser, thus reducing the Reynolds number by a factor of 3. Flows with four downstream Reynolds numbers, namely 1250, 1730, 1960 and 2500, were examined in detail. On the basis of figure 12 it will appear therefore that the flow upstream of the diffuser in all four cases was fully turbulent. This was also shown to be the case by Badrinarayanan. Detailed measurements of turbulence intensity and shear stresses showed that the flow downstream of the diffuser was reverting, asymptotically, to the laminar state in all the four cases which were examined. The decay of turbulence, with distance from the end of the diffuser, was found to be faster in the case of the flow with the lowest Reynolds number. An extrapolation of these rates of decay to zero suggests that reverse transition will not occur for downstream Reynolds numbers greater than about 2900. Considering the difficulties and uncertainties in making such measurements, this figure compares extremely well with the value of $Re = 3000$ suggested earlier as being the lowest Reynolds number for which fully turbulent flow can be maintained in a two-dimensional channel. Thus, the measurements of Badrinarayanan provide a further and much more direct check on the proposed criterion for the onset of reverse transition.

5. Conclusions

The available experimental evidence strongly suggests that initial breakdown of fully developed turbulent flow, as evidenced by departures from the inner-law velocity distribution or from the appropriate skin-friction laws, occurs when the shear-stress distribution in the wall region takes on a form typical of a strong favourable pressure gradient. This has been demonstrated experimentally for boundary-layer flow and is shown to be compatible with the results for axisymmetric pipe flow and parallel flow in a two-dimensional channel.

The suggestion that the distribution of shear stress in the wall region rather than the overall Reynolds number is the important parameter in initiating reverse transition does not appear to be in conflict with known results. In particular, it is not at variance with Preston's (1958) observation that a turbulent boundary layer in zero pressure gradient cannot persist unless the boundary-layer Reynolds number, R_θ, exceeds a certain minimum value; all that is suggested is that where a fully developed turbulent flow *already exists* subsequent breakdown can occur at quite high Reynolds numbers if the shear stress distribution imposed on the flow in the neighbourhood of the wall is typical of a strong favourable pressure gradient.

The value of the shear stress gradient parameter $\Delta_\tau = \nu\alpha/\rho U_\tau^3$ associated with the breakdown of fully developed turbulent flow appears to lie in the region of $-0{\cdot}009$. It cannot be stated with greater precision at this stage for several reasons: in the case of the turbulent boundary layer the flow accelerations can be significant and therefore, in the absence of direct measurements, shear-stress distributions have had to be inferred from a mixing-length analysis; in the case of pipe flow the point of departure from the usual skin-friction law is not very well defined and for the parallel channel flow more experiments are required to establish a more precise value for the minimum Reynolds number for which fully turbulent flow can be maintained. In view of these uncertainties the critical value of Δ_τ quoted here must be regarded as tentative. Further experiments of a more critical nature are now being carried out and it is hoped to report these later on.

It will be obvious that, whatever the conditions leading to the breakdown of the fully developed turbulent flow, the Reynolds number of the final laminar flow is unlikely to exceed that for which the laminar flow is formally stable. It would be of interest to examine the characteristics of intermediate states of flow between fully developed turbulent and completely laminar and to see whether in suitable conditions they can be maintained unaltered in the downstream direction. With the recent flow visualization studies of Schraub & Kline (1965), hot-wire measurements of Laufer (1962), Sibulkin (1962), Launder (1964), Launder & Stinchcombe (1967) and Badrinarayanan (1966), and the intermittency measurements of Fiedler & Head (1966) it is now possible to form a fairly clear physical picture of the rather complicated process of reverse transition. A discussion of the complete process is, however, beyond the scope of the present paper.

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